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Hagedorn Thermostat: A Novel View of Hadronic Thermodynamics

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A microcanonical treatment of Hagedorn systems, i.e. finite mass hadronic resonances with an exponential mass spectrum controlled by the Hagedorn temperature T_H, is performed. We show that, in the absence of any restrictions, a Hagedorn system is a perfect thermostat, i.e. it imparts its temperature T_H to any other system in thermal contact with it. We study the thermodynamic effects of the lower mass cut-off in the Hagedorn mass spectrum. We show that in the presence of a single Hagedorn resonance the temperature of any number of N_B Boltzmann particles differs only slightly from T_H up to the kinematically allowed limit N_B^{kin}. For $N_B > N_B^{kin}$ however, the low mass cut-off leads to a decrease of the temperature as N_B grows. The properties of Hagedorn thermostats naturally explain a single value of hadronization temperature observed in elementary particle collisions at high energies and lead to some experimental predictions.

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I. INTRODUCTION

The statistical bootstrap model (SBM) [1, 2] gave the first evidence that an exponentially growing hadronic mass spectrum $g_H(m) = \exp[m/T_H]\ (m_{\rm o}/m)^a$ for $m \rightarrow \infty$ (the constants $m_{\rm o}$ and a will be defined later) could lead to new thermodynamics above the Hagedorn temperature T_H. Originally, the divergence of thermodynamic functions at temperatures T above T_H was interpreted as the existence of a limiting temperature for hadrons. In other words, it is impossible to build the hadronic thermostat above T_H. A few years later an exponential form of the asymptotic mass spectrum was found in the MIT bag model [3] and the associated limiting temperature was interpreted as the phase transition temperature to the partonic degrees of freedom [4]. These results initiated extensive studies of hadronic thermodynamics within the framework of the gas of bags model (GBM) [5, 6]. The SBM with a non-zero proper volume [7] of hadronic bags was solved analytically [8] by the Laplace transform to the isobaric ensemble and the existence of phase transition from hadronic to partonic matter (also called the quark gluon plasma, QGP) was shown. Since then this technique has been used to solve more sophisticated versions [9, 10] of the SBM and other statistical models [11].

The major achievement of the SBM is that it naturally explains why the temperature of secondary hadrons created in hadronic collisions cannot exceed T_H. However, this result is based on two related assumptions. First, the grand canonical formulation for SBM is appropriate, and second, the resonances of infinite mass should contribute to thermodynamic functions.

Very recently, using the microcanonical formulation, we showed [12] that in the absence of any restrictions on the mass, resonances with the Hagedorn mass spectrum behave as a perfect thermostat and perfect chemical reservoir, i.e. they impart the Hagedorn temperature T_H to particles which are in thermal contact and force them to be in chemical equilibrium. Similar questions about chemical properties of the heavy resonances were addressed in [13].

Our analysis [12] based on very general thermodynamic arguments shows that *it is improper to include any temperature other than* T_H into canonical and/or grand canonical formulations of the statistical mechanics of any system coupled to a Hagedorn thermostat. We also demonstrated that the Hagedorn thermostat generates a volume independent concentration of the particles in chemical equilibrium with it [12]. Thus, the entire framework of the SBM and GBM, which is also based on these two assumptions, must be revisited. In other words, it is necessary to return to the foundations of the statistical mechanics of hadrons and study the role of the Hagedorn mass spectrum for finite masses of hadronic resonances above the cut-off value $m_{\rm o}$, below which the hadron mass spectrum is discrete. Such an analysis for an arbitrary value of a in $g_H(m)$ was not done in [12], and, will be performed here.

This refinement is important for understanding the differences and similarities between A+A and elementary particle collisions at high energies. There are two temperatures measured in A+A collisions that are very close to the transition temperature T_{Tr} from hadron gas to QGP calculated from the lattice quantum chromodynamics [14] at vanishing baryonic density. The first is the chemical freeze-out temperature at vanishing baryonic density $T_{Chem} \approx 175 \pm 10$ MeV of the most abundant hadrons (pions, kaons, nucleons *etc*) extracted from particle multiplicities at highest SPS [15] and all RHIC [16] energies. Within the error bars $T_{Tr} \approx T_{Chem}$ is also very close to the kinetic freeze-out temperature T_{Kin} (i.e. hadronization temperature) found from the transverse mass spectra of heavy, weakly interacting hadrons such as Ω hyperons, J/ψ and ψ' mesons at the highest SPS energy [17, 18], and Ω-hyperons [19, 20, 21] and ϕ meson [19, 21, 22] at $\sqrt{s} = 130$ A·GeV and $\sqrt{s} = 200$ A·GeV energies of RHIC. The existence of the deconfinement transition naturally explains the same value for all

these temperatures.

Can the same logic be applied to the collisions of elementary particles, where the formation of a deconfined quark gluon matter is rather problematic? The fact that the hadronization temperature [23] and inverse slopes of the transverse mass spectra of various hadrons [24] found in elementary particle collisions at high energies are similar to those ones for A+A collisions is tantalizing.

In the present paper we show that the results of A+A and elementary particle collisions can be understood and explained on the same footing by the Hagedorn thermostat concept. For this purpose we study the properties of microcanonical equilibrium of Boltzmann particles which are in contact with Hagedorn thermostats and elucidate the effect of the mass cut-off $m_{\rm o}$ of the Hagedorn spectrum on the temperature of a system at given energy. The microcanonical formulation of the Hagedorn thermostat model (HTM) is given in Section II. Section III is devoted to the analysis of the most probable state of a single heavy Hagedorn thermostat of mass $m \geq m_{\rm o}$ and N_B Boltzmann particles. The last section contains our conclusions and possible experimental consequences.

II. HAGEDORN THERMOSTAT MODEL

Let us consider the microcanonical ensemble of N_B Boltzmann point-like particles of mass m_B and degeneracy g_B, and N_H hadronic point-like resonances of mass m_H with a mass spectrum $g_H(m_H) = \exp[m_H/T_H](m_{\rm o}/m_H)^a$ for $m_H \geq m_{\rm o}$ which obeys the inequalities $m_{\rm o} \gg T_H$ and $m_{\rm o} > m_B$. A recent analysis [25] suggests that the Hagedorn mass spectrum can be established for $m_{\rm o} < 2$ GeV.

In the SBM [26] and the MIT bag model [5] it was found that for $m_H \to \infty$ the parameter $a \leq 3$. For finite resonance masses the value of a is unknown, so it will be considered as a fixed parameter.

The microcanonical partition of the system, with volume V, total energy U and zero total momentum, can be written as follows

$$\Omega = \frac{V^{N_H}}{N_H!}\left[\prod_{k=1}^{N_H} g_H(m_H)\int\frac{d^3Q_k}{(2\pi)^3}\right]\frac{V^{N_B}}{N_B!}\left[\prod_{l=1}^{N_B} g_B\int\frac{d^3p_l}{(2\pi)^3}\right]$$

$$\delta\Bigg(U - \sum_{i=1}^{N_H}\epsilon_i^H - \sum_{j=1}^{N_B}\epsilon_j^B\Bigg), \qquad (1)$$

where the quantity $\epsilon_i^H = \varepsilon(m_H, Q_i)$ $\big(\epsilon_j^B = \varepsilon(m_B, p_j)$ and $\varepsilon(M,P) \equiv \sqrt{M^2+P^2}\big)$ denotes the energy of the Hagedorn (Boltzmann) particle with the 3-momentum $\vec{Q}_i$ ($\vec{p}_j$). In order to simplify the presentation of our idea, Eq. (1) accounts for energy conservation only and neglects momentum conservation.

The microcanonical partition (1) can be evaluated by the Laplace transform in total energy U [27]. Then the momentum integrals in (1) are factorized and can be performed analytically. The inverse Laplace transform in the conjugate variable λ can be done analytically for the nonrelativistic and ultrarelativistic approximations of the one-particle momentum distribution function

$$\int_0^\infty\frac{d^3p\; e^{-\lambda\varepsilon(M,p)}}{(2\pi)^3} \approx \begin{cases} \left[\frac{2M}{\lambda}\right]^{\frac{3}{2}} I_{\frac{1}{2}} e^{-M\lambda}\,, & MRe(\lambda) \gg 1\,, \\ \frac{2}{\lambda^3}\; I_2\,, & MRe(\lambda) \ll 1\,. \end{cases} \qquad (2)$$

where the auxiliary integral is denoted as

$$I_b \;\equiv\; \int_0^\infty \frac{d\xi}{(2\pi)^2}\;\xi^b\;e^{-\xi}\,. \qquad (3)$$

Since the formal steps of further evaluation are similar for both cases, we discuss in detail the nonrelativistic limit only, and later present the results for the other case. The nonrelativistic approximation ($MRe(\lambda) \gg 1$) for Eq. (1) is as follows

$$\Omega_{nr} = \frac{\left[V g_H(m_H)\,[2m_H]^{\frac{3}{2}}\, I_{\frac{1}{2}}\right]^{N_H}}{N_H!}$$
$$\frac{\left[V g_B\,[2m_B]^{\frac{3}{2}}\;\; I_{\frac{1}{2}}\right]^{N_B}}{N_B!}\;\frac{E_{kin}^{\frac{3}{2}(N_H+N_B)-1}}{\left(\frac{3}{2}(N_H+N_B)-1\right)!}\,, \qquad (4)$$

where $E_{kin} = U - m_H N_H - m_B N_B$ is the kinetic energy of the system.

As shown below, the most realistic case corresponds to the nonrelativistic treatment of the Hagedorn resonances because the resulting temperature is much smaller than their masses. Therefore, it is sufficient to consider the ultrarelativistic limit for the Boltzmann particles only. In this case ($MRe(\lambda) \ll 1$) the equation (1) can be approximated as

$$\Omega_{ur} = \frac{\left[V g_H(m_H)\,[2m_H]^{\frac{3}{2}}\;\; I_{\frac{1}{2}}\right]^{N_H}}{N_H!}$$
$$\frac{[V g_B\;2\;I_2]^{N_B}}{N_B!}\;\frac{E_{kin}^{\frac{3}{2}(N_H+2N_B)-1}}{\left(\frac{3}{2}(N_H+2N_B)-1\right)!}\,, \qquad (5)$$

where the kinetic energy does not include the rest energy of the Boltzmann particles, i.e. $E_{kin} = U - m_H N_H$.

Within our assumptions the above results are general and can be used for any number of particles, provided $N_H + N_B \geq 2$. It is instructive to consider first the simplest case $N_H = 1$. This oversimplified model, in which a Hagedorn thermostat is always present, allows us to study the problem rigorously. For $N_H = 1$ we treat the mass of Hagedorn thermostat m_H as a free parameter and determine the value which maximizes the entropy of the system. The solution $m_H^* > 0$ of

$$\frac{\delta \ln\Omega_{nr}(N_H=1)}{\delta\; m_H} \;=$$
$$\frac{1}{T_H} \;+\; \left(\frac{3}{2} \;-\; a\right)\frac{1}{m_H^*} \;-\; \frac{3(N_B+1)}{2\;E_{kin}} \;=\; 0 \qquad (6)$$

provides the maximum of the system's entropy, if for $m_H = m_H^*$ the second derivative is negative

$$\frac{\delta^2 \ln \Omega_{nr}(N_H = 1)}{\delta\ m_H^2} = - \left(\tfrac{3}{2}\ -\ a\right) \tfrac{1}{{m_H^*}^2}\ -\ \tfrac{3(N_B+1)}{2\ E_{kin}^2}\ <\ 0\,. \qquad (7)$$

If the inequality (7) is satisfied, then the extremum condition (6) defines the temperature of the system of (N_B+1) nonrelativistic particles

$$T^*(m_H^*) \equiv \frac{2\ E_{kin}}{3(N_B+1)} = \frac{T_H}{1\ +\ \left(\frac{3}{2}\ -\ a\right) \frac{T_H}{m_H^*}}\,. \qquad (8)$$

Thus, as $m_H^* \to \infty$ it follows that $T^*(m_H^*) \to T_H$, while for finite $m_H^* \gg T_H$ and $a > \frac{3}{2}$ ($a < \frac{3}{2}$) the temperature of the system is slightly larger (smaller) than the Hagedorn temperature, i.e. $T^* > T_H$ ($T^* < T_H$). Formally, the temperature of the system in equation (8) may differ essentially from T_H for a light thermostat, i.e. for $m_H^* \le T_H$. However, it is assumed that the Hagedorn mass spectrum exists above the cut-off mass $m_o \gg T_H$, thus $m^* \gg T_H$.

III. THE ROLE OF THE MASS CUT-OFF

Now we study the effect of the mass cut-off of the Hagedorn spectrum on the inequality (7) in more detail. For $a \le \frac{3}{2}$ the condition (7) is satisfied. For $a > \frac{3}{2}$ the inequality (7) is equivalent to

$$\frac{{m_H^*}^2}{\left(a - \frac{3}{2}\right)\ T^*(m_H^*)}\ >\ \frac{3}{2}\ (N_B+1)\ T^*(m_H^*)\,, \qquad (9)$$

which means that a Hagedorn thermostat should be massive compared to the kinetic energy of the system.

A more careful analysis shows that for a negative value of the determinant D_{nr} ($\tilde N \equiv N_B - \frac{2}{3}a$)

$$D_{nr}\ \equiv\ \left(U - m_B N_B - \tfrac{3}{2}\ T_H\ \tilde N\right)^2 - 4\left(a - \tfrac{3}{2}\right)\ T_H\ \left(U - m_B N_B\right)\ <\ 0\,, \qquad (10)$$

equation (6) has two complex solutions, while for $D_{nr}\ =\ 0$ there exists a single real solution of (6). Solving (10) for $(U - m_B N_B)$, shows that for $\tilde N > \frac{2}{3}a - 1$, i.e. for $N_B > \frac{4}{3}, a - 1$ the inequality (10) does not hold and $D_{nr} > 0$. Therefore, in what follows we will assume that $N_B > \frac{4}{3}a - 1$ and only analyze the case $D_{nr} > 0$. For this case equation (6) has two real solutions

$$m_H^\pm = \tfrac{1}{2}\left[U - m_B N_B - \tfrac{3}{2}\ T_H\ \tilde N\ \pm\ \sqrt{D_{nr}}\right]\,. \qquad (11)$$

For $a \le \frac{3}{2}$ only m_H^+ solution is positive and corresponds to a maximum of the microcanonical partition Ω_{nr}.

For $a > \frac{3}{2}$ both solutions of (6) are positive, but only m_H^+ is a maximum. From the two limiting cases:

$$\frac{\delta \ln \Omega_{nr}(N_H = 1)}{\delta\ m_H}\ \approx\ \left(\tfrac{3}{2} - a\right) \tfrac{1}{m_H} \quad \text{for} \quad m_H \approx 0\,, \qquad (12)$$

$$\frac{\delta \ln \Omega_{nr}(N_H = 1)}{\delta\ m_H}\ \approx\ \tfrac{3(N_B+1)}{2\ E_{kin}} \quad \text{for} \quad E_{kin} \approx 0\,, \qquad (13)$$

and the fact that $m_H^\pm$ obey the inequalities

$$0\ <\ m_H^-\ \le\ m_H^+\ <\ U - m_B N_B\,, \qquad (14)$$

it is clear that $m_H^* = m_H^-$ is a local minimum of the microcanonical partition Ω_{nr}, while $m_H^* = m_H^+$ is a local maximum of the partition Ω_{nr}.

Using Eq. (11) for m_H^+, it is clear that for any value of a the constraint $m_H^+ \ge m_o$ is equivalent to the inequality

$$N_B\ \le\ N_B^{kin} \equiv \frac{U\ -\ \left[\frac{m_o}{T_H}\ -\ a\right]\ T^*(m_o)}{m_B\ +\ \frac{3}{2}\ T^*(m_o)}\,. \qquad (15)$$

Thus, at fixed energy U for all $N_B \le N_B^{kin}$ at $m_H^* = m_H^+$ there is a local maximum of the microcanonical partition Ω_{nr} with the temperature $T = T^*(m_H^+)$. For $N_B > N_B^{kin}$ the maximum of the partition Ω_{nr} cannot be reached due to the cut-off constraint and, consequently, the most probable state corresponds to $m_H = m_o$ with $T \le T^*(m_o)$ from Eq. (8). In other words, for $N_B > N_B^{kin}$ the amount of energy U is insufficient for the mass of the Hagedorn thermostat to be above the cut-off m_o and simultaneously maintain the temperature of the Boltzmann particles according to Eq. (8). By assumption there is a single Hagedorn thermostat in the system, therefore, as N_B grows the temperature of the system decreases from $T^*(m_o)$ value. Thus, the equality (15) defines the kinematical limit for reaching the maximum of the microcanonical partition.

To prove that the maximum of the microcanonical partition at $m_H = m_H^+$ is global it is sufficient to show that the constraint $m_H^+ \ge m_o$ is not consistent with the condition $m_H^- > m_o$. For $a \le \frac{3}{2}$ the maximum is global because for $0 < m_H < m_H^+$ ($m_H > m_H^+$) the partition $\Omega_{nr}(N_H = 1, m_H)$ monotonically increases (decreases) with m_H. For $a > \frac{3}{2}$ it is clear that the maximum at $m_H = m_H^+$ is local, if the state with mass $m_H = m_o$ is more probable, i.e. $\Omega_{nr}(N_H = 1, m_o) > \Omega_{nr}(N_H = 1, m_H^+)$. Due to (14) this can occur, if $m_H^- > m_o$. Substituting Eq. (11) into the last inequality, shows that this inequality reduces to the condition $N_B > N_B^{kin}$. This contradicts the constraint $m_H^+ \ge m_o$ in the form of Eq. (15). Thus, the maximum of the microcanonical partition is global.

To complete our consideration of the nonrelativistic case let us express the partition (4) in terms of the temperature (8). Applying the Stirling approximation to the factorial $(\frac{3}{2}(N_B+1) - 1)!$ for $N_B^{kin} > N_B \gg 1$ and reversing the integral representations (2) and (3) for $\lambda = 1/T^*(m_H^+)$, one finds

$$\Omega_{nr}(N_H = 1)\ =\ \frac{V\, g_H(m_H^+)}{T^*(m_H^+)} \int \frac{d^3Q}{(2\pi)^3}\ e^{-\frac{\sqrt{{m_H^+}^2 + Q^2}}{T^*(m_H^+)}} \frac{e^{\frac{U}{T^*(m_H^+)}}}{N_B!} \left[V\, g_B \int \frac{d^3p}{(2\pi)^3}\ e^{-\frac{\sqrt{m_B^2 + p^2}}{T^*(m_H^+)}}\right]^{N_B}\,. \qquad (16)$$

This is just the grand canonical partition of $(N_B + 1)$ Boltzmann particles with temperature $T^*(m_H^+)$. If $N_B > N_B^{kin} \gg 1$, then $T^*(m_H^+)$ in (16) should be replaced by $T_o(N_B) \equiv \frac{2(U - m_B N_B - m_o)}{3(N_B+1)}$.

Fig. 1 shows that for $a > \frac{3}{2}$ the system's temperature $T = T^*(m_H^+)$ as a function of N_B remains almost constant for $N_B < N_B^{kin}$, reaches a maximum at N_B^{kin} and rapidly decreases like $T = T_o(N_B)$ for $N_B > N_B^{kin}$. For $a < \frac{3}{2}$ the temperature has a plateau $T = T^*(m_H^+)$ for $N_B < N_B^{kin}$, and rapidly decreases for $N_B > N_B^{kin}$ according to $T_o(N_B)$.

The same results are valid for the ultrarelativistic treatment of Boltzmann particles. Comparing the nonrelativistic and ultrarelativistics expressions for the microcanonical partition, i.e. equations (4) and (5), respectively, one finds that the derivation of the ultrarelativistic limit requires only the substitution $N_B \rightarrow 2N_B$ and $m_B/T_H \rightarrow 0$ in equations $(6 - 16)$. Note that this substitution does not alter the expression for the temperature of the system, i.e. the right hand side of (8).

Finally, we show that for a heavy Hagedorn thermostat $(m_H^+ \gg m_o)$ these results remain valid for a single Hagedorn thermostat split into N_H pieces of the same mass. Substituting $m_H \rightarrow m_H N_H$ in the nonrelativistic expressions (4) and minimizing it with respect to m_H, the temperature of the system in the form of equation (8) is $T^*(m_H^* N_H)$, where the mass of N_H Hagedorn thermostats m_H^* is related to the solution m_H^+ of equation (11) as $m_H^* = m_H^+/N_H$. Since the original single thermostat of mass m_H^+ was assumed to be heavy, it follows $T^*(m_H^* N_H) = T^*(m_H^+) \rightarrow T_H$. A more careful study (see also [12]) using an exact expression for the microcanonical partition of N_H Hagedorn thermostats of the same mass m_H gives the same result, if $m_H \gg m_o$. A generalization of these statements to the case of N_H heavy Hagedorn thermostats of different masses also leads to the same result. Thus, splitting a single heavy Hagedorn thermostat into an arbitrary number of heavy resonances (heavier than m_o) does not change the temperature of the system.

IV. CONCLUSIONS

In the present work we generalized the SBM results [26] to systems of finite energy by showing explicitly that even a single resonance with the Hagedorn mass spectrum degeneracy, i.e. *a Hagedorn thermostat,* keeps an almost constant temperature close to T_H for any number of Boltzmann particles $3 < N_B \leq N_B^{kin}$. For the high energy limit $U \gg m_o$ this means that a single Hagedorn resonance defines the temperature of the system to be only slightly different from T_H until the energy of the Hagedorn thermostat is almost negligible compared to U. In contrast to the grand canonical formulation of the original SBM [26], in the presence of a Hagedorn thermostat the temperature T_H can be reached at any energy density.

The thermostatic nature of a Hagedorn system obviously explains the ubiquity of both the inverse slopes of measured transverse mass spectra [24] and hadronization temperature found in numerical simulations of hadrons created in elementary particle collisions at high energies [23, 28, 29]. By a direct evaluation of the microcanonical partition we showed that in the presence of a single Hagedorn thermostat the energy spectra of particles become exponential with no additional assumptions, e.g. *phase space dominance* [30] or *string tension fluctuations* [31]. Also the limiting temperature found in the URQMD calculations made in a finite box [32] can be explained by the effect of the Hagedorn thermostat. We expect that, if the string parametrization of the URQMD in a box [32] was done microcanonically instead of grand canonically, then the same behavior would be found.

The Hagedorn thermostat model generalizes the statistical hadronization model which successfully describes the particle multiplicities in nucleus-nucleus and elementary collisions [23, 28, 29]. The statistical hadronization model accounts for the decay of heavy resonances (clusters in terms of Refs. [23, 28, 29]) only and does not consider the additional particles, e.g. light hadrons, free quarks and gluons, or other heavy resonances. As we showed, the splitting of a single heavy Hagedorn resonance into several does not change the temperature of the system. This finding justifies the main assumption of the canonical formulation of the statistical hadronization model [28] that smaller clusters may be reduced to a single large cluster. Thus, recalling the MIT Bag model

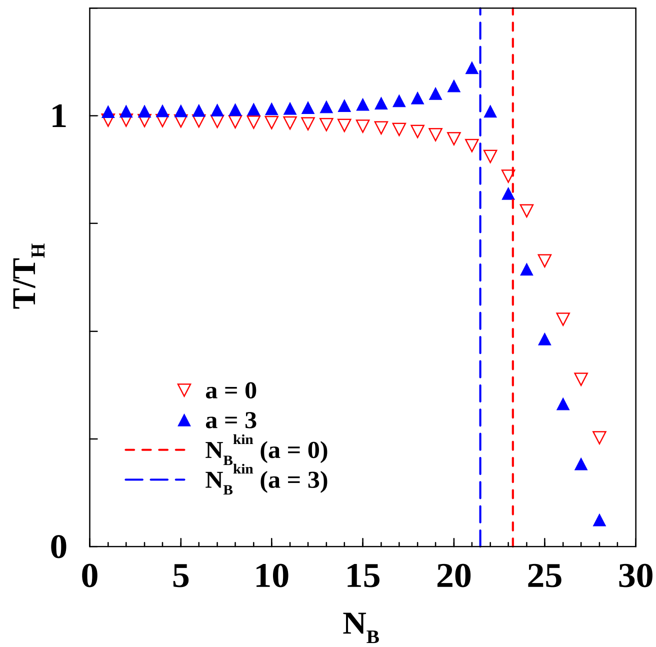

FIG. 1: A typical behavior of the system's temperature as the function of the number of Boltzmann particles N_B for $a = 3$ and $a = 0$ for the same value of the total energy $U = 30\,m_B$. Due to the thermostatic properties of a Hagedorn resonance the system's temperature is nearly constant up to the kinematically allowed value N_B^{kin} given by (15).

interpretation of the Hagedorn mass spectrum [5, 6], we conclude that quark gluon matter confined in heavy resonances (hadronic bags) controls the temperature of surrounding particles close to T_H, and, therefore, this temperature can be considered as a coexistence temperature for confined color matter and hadrons. Moreover, as we showed, the emergence of a coexistence temperature does not require the actual deconfinement of the color degrees of freedom, which, in terms of the GBM [8], is equivalent to the formation of the infinitely large and infinitely heavy hadronic bag.

Within the framework of the Hagedorn thermostat model we found that even for a single Hagedorn thermostat and $a > \frac{3}{2}$ the system's temperature $T = T^*(m_H^+)$ as a function of N_B remains almost constant for $N_B < N_B^{kin}$, reaches a maximum at N_B^{kin} and rapidly decreases for $N_B > N_B^{kin}$ (see Fig. 1). For $a < \frac{3}{2}$ the temperature has a plateau $T = T^*(m_H^+)$ for $N_B < N_B^{kin}$, and rapidly decreases for $N_B > N_B^{kin}$. If such characteristic behavior of the hadronization temperature or the hadronic inverse slopes can be measured as a function of event multiplicity, it may be possible to experimentally determine the value of a. For quantitative predictions it is necessary to include more hadronic species into the model, but this will not change our result.

If we apply the HTM to elementary particle collisions at high energy, then, as shown above, the temperature of created particles will be defined by the most probable mass of the Hagedorn thermostat. If the most probable resonance mass grows with the energy of collision, then the hadronization temperature should decrease (increase) to T_H for $a > \frac{3}{2}$ ($a < \frac{3}{2}$). Such a decrease is observed in reactions of elementary particles at high energies, see Table 1 in Ref. [29].

In order to apply these results in a more physical fashion to the quark gluon plasma formation in relativistic nucleus-nucleus collisions (where the excluded volume effects are known to be important [7, 8, 15, 33] for all hadrons) the eigen volumes of all particles should be incorporated into the model. For pions this should be done in relativistic fashion [34]. Also the effect of finite width of Hagedorn resonances may be important [10] and should be studied.

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